

THE UNIVERSITY  
*of* LIVERPOOL

Degree of Bachelor of Science : Year 3  
Degree of Master of Physics : Year 3

**ADVANCED OBSERVATIONAL ASTRONOMY**

TIME ALLOWED : Three Hours

---

INSTRUCTION TO CANDIDATES

Answer **all** questions.

Question 1 carries 40% of the total marks.

Questions 2 and 3 each carry 30% of the total marks.

The marks allotted to each part of a question are indicated in square brackets.

In the event of a student answering both parts of an either/or question and not clearly crossing out one answer, only the answer to part (a) of the question will be marked.

---

You are allowed to quote the following relation without proof:

The Planck function is

$$B_{\nu}(T) = \frac{2h\nu^3}{c^2} \frac{1}{(e^{h\nu/kT} - 1)}$$

THE UNIVERSITY  
*of* LIVERPOOL

Section A

1. (a) Show that material falling freely from a large distance directly onto the surface of a typical White Dwarf star (mass  $M = 1 M_{\odot}$ , radius  $R = 0.01 R_{\odot}$ ) will reach temperatures high enough to produce strong thermal X-ray emission. Why are observed gas temperatures in accreting White Dwarf stars somewhat lower than this? [6]
- (b) Discuss whether you would be able to detect the Zeeman components of a  $Fe^{56}$  line at wavelength  $\lambda_0 = 4000$  Angstrom, at a temperature of 6000 K, with a magnetic field of 1000 Gauss (the natural half-width of the line is negligible). [10]
- (c) Explain the key physical reason for the design of infrared telescopes being modified from that of telescopes operating in visible bands. Write brief notes on four specific modifications that are used. [6]
- (d) Discuss whether or not you can observe absorption lines in the spectrum of an isothermal gas in thermodynamic equilibrium [6]
- (e) Given that all but 0.01% of the molecular gas in the Universe is  $H_2$ , explain why it is necessary to observe the spectral lines of molecules such as CO when investigating cold molecular clouds in the interstellar medium. Why is CO itself particularly useful? [6]
- (f) Explain why it is desirable to operate any detector in the regime in which it is 'background-noise-limited', and what this phrase means. [6]

THE UNIVERSITY  
*of* LIVERPOOL

Section B

2. Answer **either** (a) **or** (b)

(a) (i) Show how the blackbody function can be used to define Brightness Temperature  $T_b$  as a measure of the strength of the radiation from an astronomical source at a particular frequency. Show that this unit of measurement is especially applicable to radio astronomy. [5]

(ii) Write down the equation of radiative transfer in terms of intensity  $I_\nu$  and explain the terms in it. Use this to derive the form in terms of  $T_b$  that is commonly used in radio astronomy. Explain how the thermodynamic temperature  $T$  of the source enters the equation and show the relationship of  $T_b$  to  $T$  differs when the radiation is optically thick and when it is optically thin. [6]

(iii) Antenna temperature  $T_A$  is a measure of the strength of signal actually detected by the receiver system and is defined by

$$kT_A \equiv \frac{1}{2}\eta A \int_{\Omega} I_\nu d\Omega.$$

Explain the terms in this expression and use it to show that, in order to maximise the signal strength from our radio detector, we ideally want the antenna beam to be the same size as the observed source. [7]

(iv) Explain the principle of the Dicke switch in radio receivers and demonstrate mathematically that switching with a matched load leads to greater stability in the detected signal than in a non-switched receiver system. [12]

THE UNIVERSITY  
*of* LIVERPOOL

2 (continued).

(b) i) What is meant by the ‘equivalent width’ of a spectral line? Show graphically the difference between the actual spectral line and its associated equivalent width. For what purpose might equivalent width be used? [6]

ii) Sketch the change of the line profile of a generic chemical species when its abundance increases, in the case of:

- 1) an optically thin line
- 2) an optically thick line

Explain your answer. [9]

iii) Suppose one observes spectroscopically a rotating star whose rotation axis is perpendicular to the line of sight (and which rotates as a solid body). Denote with  $\lambda$  the geographical longitude on the star disk counted from the central meridian,  $\phi$  the geographical latitude,  $R$  the stellar radius,  $\omega$  the angular velocity and  $V_{\text{rot}}=R \omega$ . Write down the relationship between the observed radial velocity of a point on the stellar disk  $V_r$ ,  $V_{\text{rot}}$ ,  $\lambda$  and  $\phi$ . On the basis of this result, derive the equation of the surfaces of constant  $V_r$  on the stellar disk. [5]

iv) Describe graphically the effect of rotation on the shape of spectral lines observed from an unresolved stellar disk. Do you expect the effect of rotation to be greater in the spectra of young open cluster stars, or in those of globular cluster stars? [7]

v) Describe the physical situation in which an individual emission line is split into two components of equal intensity, and the emitted photons have circular polarization. [3]

THE UNIVERSITY  
*of* LIVERPOOL

3. Answer **either** (a) **or** (b)

(a) (i) Describe, with figures, two types of system used for focusing in X-Rays, one appropriate for low-energy photons and one for high-energy photons. Explain why different methods are needed in these two regimes. [8]

(ii) A diffraction grating with line spacing  $d$  is used at grazing incidence for X-ray spectroscopy. If the grazing angle is  $\theta$  and the angle of reflection is  $\theta + \phi$ , and both are small, use a Taylor expansion in  $\phi$  to show that the path difference between adjacent reflected rays is

$$\Delta P \simeq \frac{1}{2}d(\phi^2 + 2\theta\phi). \quad [6]$$

Hence show that spectra of order  $m$  at wavelength  $\lambda$  are observed at angles

$$\phi = \left( \frac{2m\lambda}{d} + \theta^2 \right)^{\frac{1}{2}} - \theta \quad [4]$$

and that the dispersion of the resulting X-ray spectrograph will be  $\sqrt{(m/2d\lambda)}$ . In what way is this different from the dispersion of a grating used for optical wavelengths? [3]

(iii) In a cluster of galaxies, a cooling flow radiating strongly at X-Ray wavelengths is found to be in thermal equilibrium.

Write down expressions for the thermal energy of  $N$  monatomic particles and for an element of work done by the incoming gas, assuming an Ideal Gas in a hydrostatic system. [2]

Use the latter to show that the work done by  $N$  particles at constant pressure is  $W = NkT$ , where  $k$  is the Boltzmann constant and  $T$  is the initial gas temperature and hence show that the luminosity of the cooling flow is

$$L = \frac{5}{2} \left( \frac{kT}{\mu m_{\text{H}}} \right) \dot{M}$$

where  $\mu$  is the average mass of the particles in the flow relative to that of the hydrogen atom ( $m_{\text{H}}$ ) and  $\dot{M}$  is the rate of mass inflow. [7]

THE UNIVERSITY  
*of* LIVERPOOL

3 (continued).

(b) i) Denote with  $q_{ij}$  the quantum efficiency of a generic pixel  $ij$  of a CCD; with  $I_{ij}$  the number of photons received by the pixel  $ij$ ; with  $X_{ij}$  the output signal from the pixel  $ij$ , and with  $G$  the transfer function of the detector. Demonstrate mathematically that, after the flat field procedure, the detected signal does not depend on  $q_{ij}$ . [6]

ii) Discuss the basic physical ideas of profile-fitting photometry. [4]

iii) Write down the analytical expression of the typical stellar PSF, and explain the physical meaning of the parameters it contains. [7]

iv) Write down the first-order Taylor expansion at a given pixel coordinate of the stellar PSF around, respectively:

$$\begin{aligned} C &= C_0 \\ B &= B_0 \\ R &= R_0 \end{aligned} \quad [8]$$

v) Describe the procedure to obtain the magnitudes of standard photometric stars [5]

## Physical Constants

EEP, November 1995

<i>Quantity</i>	<i>Symbol</i>	<i>Value</i>	<i>Units</i>
Speed of light	$c$	$2.998 \times 10^8$	$\text{m s}^{-1}$
Permeability of free space	$\mu_0$	$4\pi \times 10^{-7}$	$\text{H m}^{-1}$
Permittivity of free space	$\epsilon_0$	$8.854 \times 10^{-12}$	$\text{F m}^{-1}$
Charge on electron	$e$	$1.602 \times 10^{-19}$	C
Planck's constant	$h$	$6.626 \times 10^{-34}$	J s
Gravitational constant	$G$	$6.673 \times 10^{-11}$	$\text{N m}^2 \text{kg}^{-2}$
Atomic mass unit	$u$	$1.660 \times 10^{-27}$	kg
Rest mass of electron	$m_e$	$9.109 \times 10^{-31}$	kg
Rest mass of proton	$m_p$	$1.673 \times 10^{-27}$	kg
Rest mass of neutron	$m_n$	$1.675 \times 10^{-27}$	kg
Boltzmann's constant	$k$	$1.381 \times 10^{-23}$	$\text{J K}^{-1}$
Avogadro's number	$N_A$	$6.023 \times 10^{23}$	$\text{mol}^{-1}$
Universal gas constant	$R$	8.314	$\text{J K}^{-1} \text{mol}^{-1}$
Stefan-Boltzmann constant	$\sigma$	$5.67 \times 10^{-8}$	$\text{W m}^{-2} \text{K}^{-4}$
Radiation constant	$a$	$7.56 \times 10^{-16}$	$\text{J m}^{-3} \text{K}^{-4}$
Bohr magneton	$\mu_B$	$9.274 \times 10^{-24}$	$\text{A m}^2$
Nuclear magneton	$\mu_N$	$5.051 \times 10^{-27}$	$\text{A m}^2$
Acceleration due to gravity	$g$	9.81	$\text{m s}^{-2}$
Standard atmospheric pressure	1 atm	$1.013 \times 10^5$	$\text{N m}^{-2}$ (Pa)
Thomson cross section	$\sigma_T$	$6.65 \times 10^{-29}$	$\text{m}^2$
Rydberg's constant for Hydrogen	$R_H$	$1.09678 \times 10^7$	$\text{m}^{-1}$

## Astronomical Constants

<i>Quantity</i>	<i>Symbol</i>	<i>Value</i>	<i>Units</i>
Earth mass	$M_\oplus$	$5.980 \times 10^{24}$	kg
Earth radius	$R_\oplus$	$6.378 \times 10^6$	m
Solar mass	$M_\odot$	$1.989 \times 10^{30}$	kg
Solar radius	$R_\odot$	$6.960 \times 10^8$	m
Solar luminosity	$L_\odot$	$3.862 \times 10^{26}$	W
Solar effective temperature	$T_{eff\odot}$	5770	K
Astronomical unit	AU	$1.496 \times 10^{11}$	m

## Conversions

<i>Quantity</i>	<i>Symbol</i>	<i>Value</i>	<i>Units</i>
Gauss	G	$10^{-4}$	T
Erg	erg	$10^{-7}$	J
Electron volt	eV	$1.602 \times 10^{-19}$	J
Parsec	pc	$3.086 \times 10^{16}$	m
		3.262	light years
Jansky	Jy	$10^{-26}$	$\text{W m}^{-2} \text{Hz}^{-1}$