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THE UNIVERSITY *of* LIVERPOOL

MAY 2004 EXAMINATIONS

Degree of Bachelor of Science : Year 3
Degree of Master of Physics : Year 3

ADVANCED OBSERVATIONAL ASTRONOMY

TIME ALLOWED : Three Hours

INSTRUCTION TO CANDIDATES

Answer **all** questions.

Question 1 carries 40% of the total marks.

Questions 2 and 3 each carry 30% of the total marks.

The marks allotted to each part of a question are indicated in square brackets.

In the event of a student answering both parts of an either/or question and not clearly crossing out one answer, only the answer to part (a) of the question will be marked.

You are allowed to quote the following relation without proof:

The Planck function is

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{(e^{h\nu/kT} - 1)}$$

Trigonometric relations:

For any arbitrary angles A and B:

$\sin(A+B) = \sin A \cos B + \cos A \sin B$
$\cos(A+B) = \cos A \cos B - \sin A \sin B$
$\sin(A-B) = \sin A \cos B - \cos A \sin B$
$\cos(A-B) = \cos A \cos B + \sin A \sin B$

For any arbitrary angle A:

$\sin^2 A + \cos^2 A = 1$
$\sin 2A = 2 \sin A \cos A$
$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

In the limit of small angle A:

$\sin A = A$
$\cos A = 1 - A^2/2$

1. (a) Describe the physical mechanism for the emission of *cyclotron* radiation. Sketch the spectrum of a cyclotron source. Name two astrophysical sources of cyclotron radiation. [**6**]

(b) Describe two types of *laser guide star* used in adaptive optics systems. Discuss the advantages and disadvantages of the two types of lasers. Explain the *tilt problem* in laser guide star systems and describe two means by which the tilt problem can be overcome. [**6**]

(c) Explain the difference between the *Michelson Stellar Interferometer* and the *Intensity Interferometer* when used for measurement of stellar diameters. Why does the latter have an advantage in terms of stability when used over baselines longer than 20 metres? [**6**]

(d) In an X-ray CCD, the number of electrons released when a photon is incident on the detector is given by $N = \frac{E}{w}$, where E is the energy of the incident photon, and w is a constant which depends upon the material. The variance on the number of electrons released is given by $\sigma_N^2 = FN$, where F is the Fano factor.
Calculate the energy resolution in electron Volts of an X-ray CCD for photons of energy 2.2keV, if w for silicon is 3.65eV, and the Fano factor for silicon is 0.12. Calculate the *resolving power*.
Discuss the physical mechanism for release of electrons by X-rays in i) CCDs, and ii) gas proportional counters. Explain why the energy resolution of the CCDs is better. [**8**]

(e) In a *semiconductor bolometer*, the resistance, R, is given by $R = R_0 exp[(T_0/T)^{0.5}]$, where T is the temperature, and T_0 and R_0 are constants which are properties of the material. Derive an expression for the temperature coefficient of resistance $\alpha = (T/R)(dR/dT)$. State two advantages of superconducting over semiconductor bolometers [**6**]

(f) What is meant by the *curve of growth* of a spectral absorption line? Sketch the curve of growth of the CaII K absorption line in main sequence stars, and identify on this sketch three regions of different dependence upon the absorber column density, briefly explaining the physics which gives rise to these dependences. [**8**]

2. Answer **either** (a) **or** (b)

(a) (i) State the equation of radiative transfer in terms of intensity, optical depth and the source function, giving definitions of the optical depth and the source function. Show by use of an integrating factor that the formal solution to the equation of radiative transfer is:

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau_\nu^{'})} S_\nu(\tau_\nu^{'}) d\tau_\nu^{'}$$

Give the particular form of the solution for the case of a source function which is uniform throughout the medium, and no background source. [**10**]

(ii) The Planck spectrum for blackbody radiation is given by:

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{(e^{h\nu/kT} - 1)}$$

Show that the frequency at which the peak of the Planck function occurs is a linear function of temperature. [**6**]

(iii) The emissivity from gas at temperature T in a cluster of galaxies radiating by thermal bremsstrahlung radiation is given by:

$$\epsilon_\nu = 6.8 \times 10^{51} Z^2 n_e^2 T^{-1/2} e^{-h\nu/kT} g(\nu, T)$$

Z is the atomic number of the gas, $g(\nu, T)$ is a slowly varying function of both ν and T. If the gas is hydrogen at 10^6 K, evaluate the frequency at which $h\nu = kT$ and thus above which the exponential term in the equation for emissivity becomes significant. [**3**]

(iv) The absorption coefficient for bremsstrahlung absorption in SI units is given by:

$$\kappa(\nu) = 1.8 \times 10^{-13} n_e^2 T^{-3/2} \nu^{-2}$$

The gas in a cluster of galaxies has a linear extent $L = 2 \times 10^{24}$ metres, uniform temperature $T = 10^6 K$, and uniform electron density $n_e = 10^3$ m^{-3}. Calculate the frequency at which the optical depth $\tau_{nu}) = 1$, and sketch the spectrum of bremsstrahlung emission from this cloud, marking the critical frequencies and showing the behaviour of the spectrum in three distinct regions defined by these frequencies. [**11**]

2 (continued).

(b) (i) Describe the principle of the *superheterodyne* radio receiver. What advantages do superheterodyne receivers give when compared with more conventional receivers? What is the difference between a superheterodyne receiver designed to operate above 40 GHz and one designed to be used at lower frequency? [**6**]

(ii) Consider a square-law mixer, i.e. one whete the output current is proportional to the square of the input voltage. Suppose that the input consists of a sky frequency and a nearby local oscillator frequency. Show that the output current includes a beat component whose frequency is the difference between the sky frequency and the local oscillator frequency, and whose amplitude is linearly proportional to the input sky signal voltage. (You will need to use the trigonometric addition relations given on the front of the exam paper). [**12**]

(iii) Describe the principles of a Phased Array, and give two advantages of phased arrays as compared to steerable dishes for the construction of low frequency radio arrays. [**8**]

(iv) Explain how the paraboloidal form of a steerable dish is maintained at a wide range of orientations in a radio telescope, and contrast this with the situation in an optical telescope. [**4**]

3. Answer **either** (a) **or** (b)

(a) (i) Explain briefly the principles of operation of a three phase Charge Coupled Device (CCD), describing in particular how photons are converted to electric charge, and how that charge is stored in and read out of the pixels of the CCD. [**12**]

(ii) The band gap between the valence and conduction bands in silicon is 1.12 electron Volts. Calculate the longest wavelength of photon which can be detected by a silicon CCD. [**4**]

(iii) Explain the advantages of the following characteristics of a CCD detector:

1. Thinned back-illuminated CCD compared with a frontside illuminated CCD. [**2**]
2. Buried Channel CCD compared with a Surface Channel CCD. [**2**]
3. Frame Transfer CCD compared with a Line Transfer CCD. [**2**]

(iv) Calculate the signal-to-noise ratio of an aperture photometry measurement of a star of magnitude V=23.5 with a CCD detector, given the following parameters:

- Equal sky and object apertures of diameter 4.0 arcseconds
- CCD pixel size 0.15 arcseconds.
- Atmospheric transmission 0.88
- Telescope aperture diameter 2.5 metres
- Telescope total efficiency 0.70
- Filter transmission 0.75
- CCD Responsive Quantum Efficiency 0.75
- Filter passband 87 nm.
- CCD readout noise 15 electrons.
- Sky brightness V = 22.5 magnitudes/square arcsecond.
- Exposure time 300 seconds.

You may assume that a star of V = 0 yields 10^8 *photons/s/*m^2*/nm at the top of the atmosphere.* [**6**]

(v) Instead of using a CCD, another astronomer uses a noise free photon counting detector with Responsive Quantum Efficiency 0.20, and pixel size 0.30 μm. Does the second astronomer obtain a better or worse signal-to-noise ratio compared with that given by the CCD detector? [**2**]

3 (continued).

(b) (i) By summing the amplitudes of rays from successive reflections between two parallel reflecting plates with a small gap d between them, show that the complex amplitude of the sum of these rays is given by:

$$Ae^{i\phi} = \frac{a(1-r^2)}{1-r^2e^{i\delta}}$$

where a is the amplitude of the incident ray, δ is the phase difference between successively reflected rays, A is the amplitude and ϕ the phase of the sum of the rays.
Then by multiplying the amplitude by its complex conjugate to obtain the intensity, show that the fringe output from a Fabry-Perot Etalon is

$$I_T = \frac{I_0}{[1+\frac{4R}{(1-R)^2}\sin^2(\frac{2\pi d\mu\cos\theta}{\lambda})]},$$

where I_T is the transmitted intensity, I_0 is a constant, R is the proportion of intensity reflected by the plates, μ is the refractive index of the material in the gap, d is the size of the gap, θ is the angle of the ray, and λ is the wavelength. [**12**]

(ii) Derive an expression for the Dispersion $\frac{d\lambda}{d\theta}$ of an etalon. [**4**]

(iii) Define the reflective finesse of an etalon, and give an expression for the reflective finesse in terms of R, the percentage of intensity reflected by the plates. Give an expression for the theoretical resolving power of the etalon in terms of the finesse. Describe two ways by which the theoretical resolving power can be optimized when designing the etalon. [**10**]

(iv) Describe two effects which degrade the performance of an etalon in practice, and result in an effective finesse which is less that the theoretical reflective finesse. [**4**]

Physical Constants

EEP, November 1995

Quantity	*Symbol*	*Value*	*Units*
Speed of light	c	2.998×10^{8}	m s^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$	H m^{-1}
Permittivity of free space	ϵ_0	8.854×10^{-12}	F m^{-1}
Charge on electron	e	1.602×10^{-19}	C
Planck's constant	h	6.626×10^{-34}	J s
Gravitational constant	G	6.673×10^{-11}	N m^{2} kg^{-2}
Atomic mass unit	u	1.660×10^{-27}	kg
Rest mass of electron	m_e	9.109×10^{-31}	kg
Rest mass of proton	m_p	1.673×10^{-27}	kg
Rest mass of neutron	m_n	1.675×10^{-27}	kg
Boltzmann's constant	k	1.381×10^{-23}	J K^{-1}
Avogadro's number	N_A	6.023×10^{23}	mol^{-1}
Universal gas constant	R	8.314	J K^{-1} mol^{-1}
Stefan-Boltzmann constant	σ	5.67×10^{-8}	W m^{-2} K^{-4}
Radiation constant	a	7.56×10^{-16}	J m^{-3} K^{-4}
Bohr magneton	μ_B	9.274×10^{-24}	A m^{2}
Nuclear magneton	μ_N	5.051×10^{-27}	A m^{2}
Acceleration due to gravity	g	9.81	m s^{-2}
Standard atmospheric pressure	1 atm	1.013×10^{5}	N m^{-2} (Pa)
Thomson cross section	σ_T	6.65×10^{-29}	m^{2}
Rydberg's constant for Hydrogen	R_H	1.09678×10^{7}	m^{-1}

Astronomical Constants

Quantity	*Symbol*	*Value*	*Units*
Earth mass	$M_\oplus$	5.980×10^{24}	kg
Earth radius	$R_\oplus$	6.378×10^{6}	m
Solar mass	$M_\odot$	1.989×10^{30}	kg
Solar radius	$R_\odot$	6.960×10^{8}	m
Solar luminosity	$L_\odot$	3.862×10^{26}	W
Solar effective temperature	$T_{eff\odot}$	5770	K
Astronomical unit	AU	1.496×10^{11}	m

Conversions

Quantity	*Symbol*	*Value*	*Units*
Gauss	G	10^{-4}	T
Erg	erg	10^{-7}	J
Electron volt	eV	1.602×10^{-19}	J
Parsec	pc	3.086×10^{16}	m
		3.262	light years
Jansky	Jy	10^{-26}	W m^{-2} Hz^{-1}